SCHISM physical formulation

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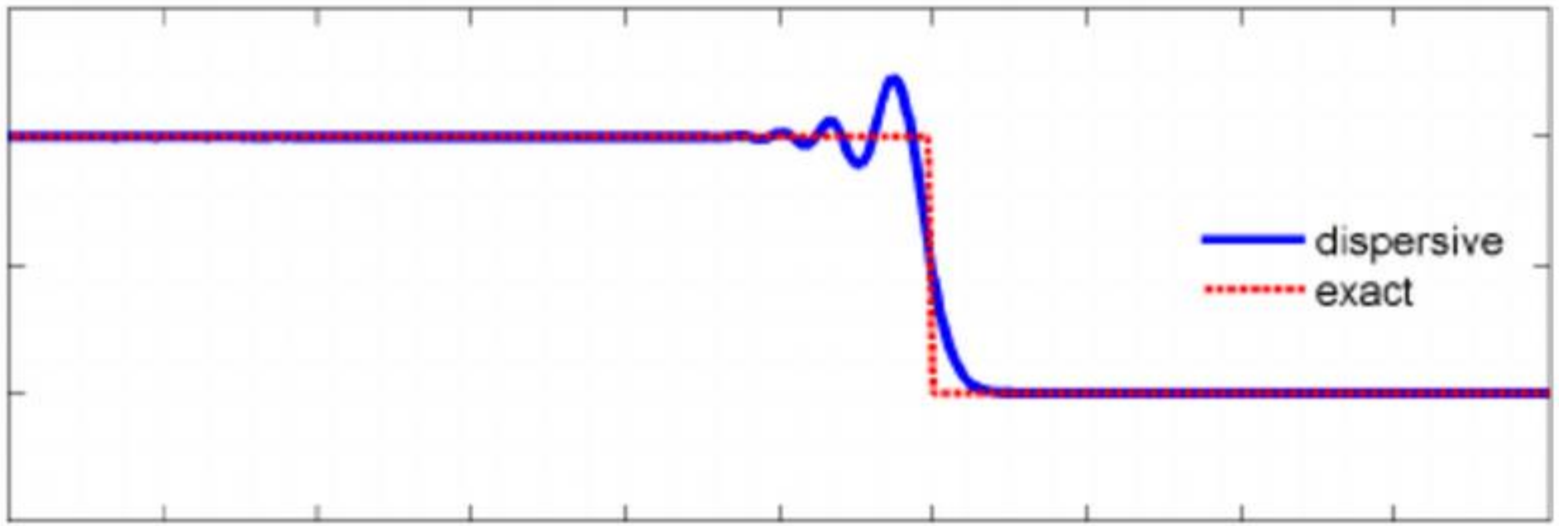
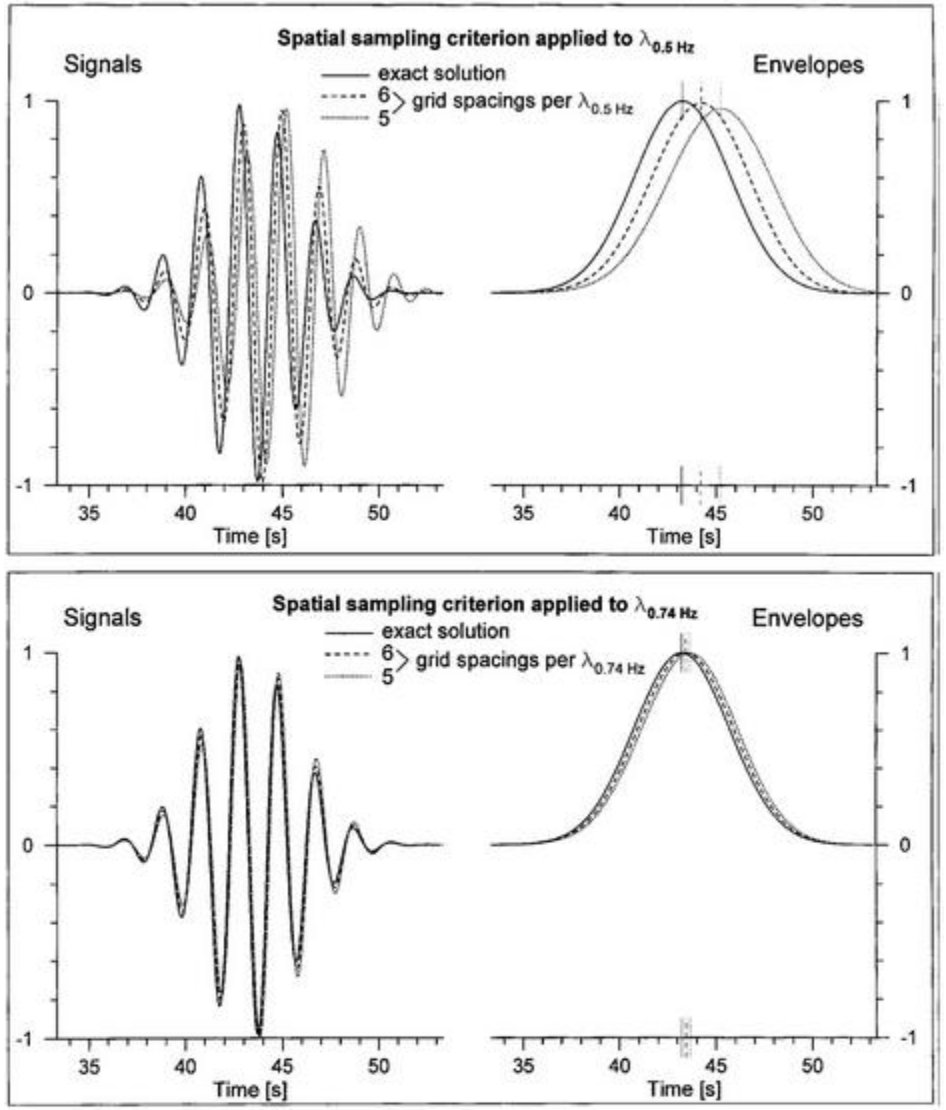
Dispersion



- Waves of different frequencies travel at different phase speeds
- Usually related to derivatives of odd order (of either physical or numerical origin)

$$\varphi_t + \varphi_{xxx} - 6\varphi\varphi_x = 0$$



With dispersion No dispersion

Governing equations: Reynolds-averged Navier-Stokes (with vegetation)

- Continuity equation

$$\nabla \cdot \mathbf{u} + \frac{\partial w}{\partial z} = 0, \quad (\mathbf{u} = (u, v))$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \int_{-h}^{\eta} \mathbf{u} dz = 0$$

- Momentum equations

vegetation

$$\frac{D\boldsymbol{u}}{dt} = \mathbf{f} - g\nabla\eta + \boldsymbol{m_z} - \alpha|\boldsymbol{u}|\boldsymbol{u}L(x,y,z)$$

$$\mathbf{f} = f(v, -u) - \frac{g}{\rho_0}\int_z^{\eta} \nabla\rho d\zeta - \frac{\nabla p_A}{\rho_0} + \alpha g \nabla\Psi + \mathbf{F}_m + other$$

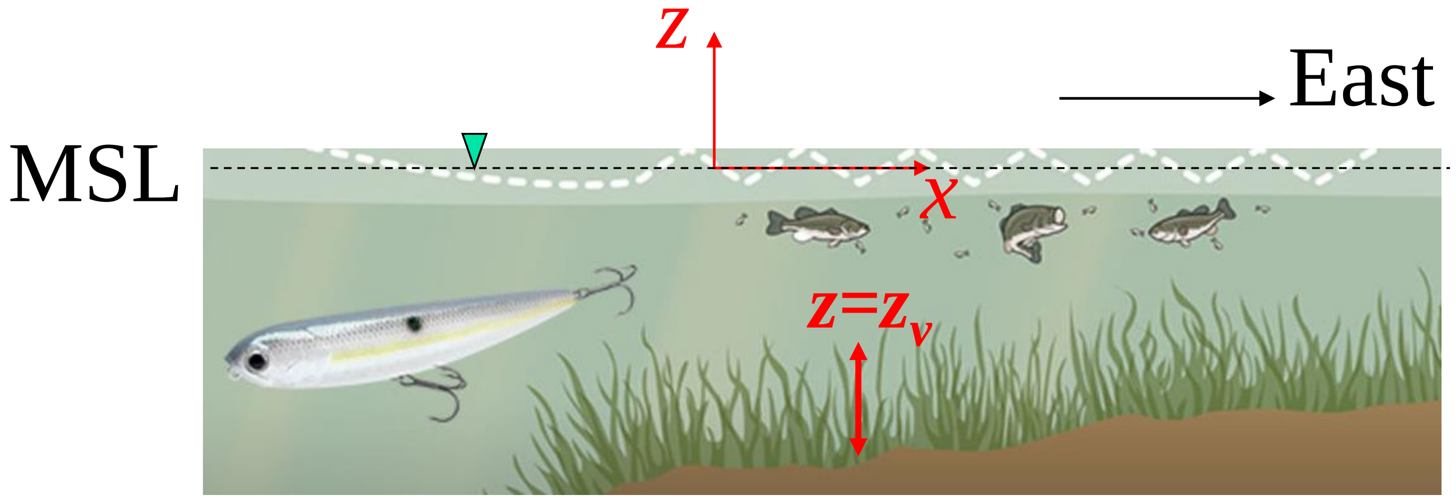
Vertical b.c. (3D):

$$\begin{cases} \nu \dfrac{\partial \boldsymbol{u}}{\partial z} = \boldsymbol{\tau}_w, & z = \eta \\ \nu \dfrac{\partial \boldsymbol{u}}{\partial z} = \chi \boldsymbol{u}_b, & z = -h \end{cases}$$

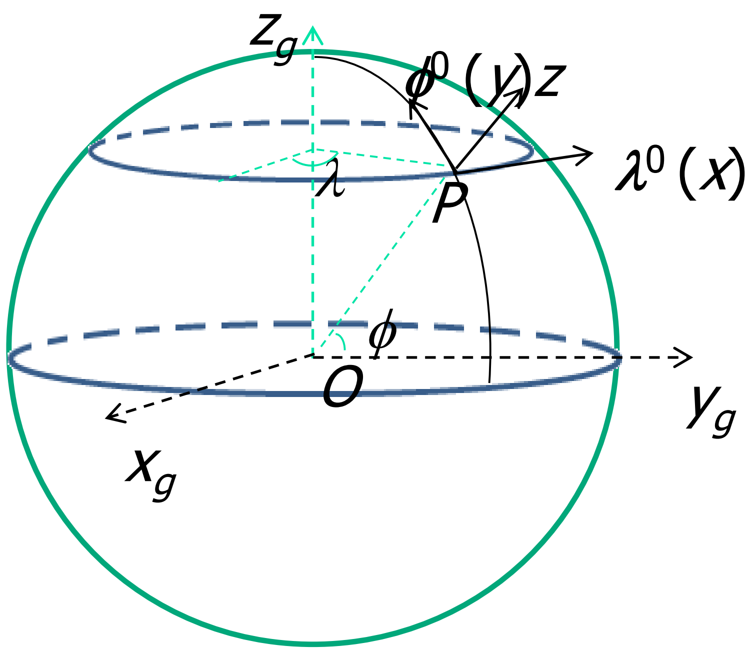
$$\boldsymbol{m}_z = \begin{cases} \dfrac{\partial}{\partial z}\left(\nu \dfrac{\partial \boldsymbol{u}}{\partial z}\right), & 3D \\ \dfrac{\boldsymbol{\tau}_w - \chi \boldsymbol{u}}{H}, & 2D \end{cases}$$

$$L(x,y,z) = \begin{cases} \mathcal{H}(z_v - z), & 3D \\ 1, & 2D \end{cases}$$

$\alpha(x,y) = D_v N_v C_{Dv}/2$ (vegetation)



Spherical coordinates



We transform the coordinates instead of eqs
There are 2 frames used:

1. Global
2. Lon/lat (local @ node/element/side)

$$\begin{cases} x_g &= R_0 \cos\phi \cos\lambda \\ y_g &= R_0 \cos\phi \sin\lambda \\ z_g &= R_0 \sin\phi \end{cases}$$

$$\begin{aligned} \boldsymbol{\lambda}^0 &= -\sin\lambda \boldsymbol{i} + \cos\lambda \boldsymbol{j} \\ \boldsymbol{\phi}^0 &= -\cos\lambda\sin\phi \boldsymbol{i} - \sin\lambda\sin\phi \boldsymbol{j} + \cos\phi \boldsymbol{k} \end{aligned}$$

- Avoids polar singularity easily
- Preserves all original matrix properties

Changes in the code is mainly related to re-project vectors and to calculate distances

Hydrostatic model

Hydrostatic assumption

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - g = 0 \Rightarrow P = g\int_z^{\eta} \rho \, d\zeta + P_A$$

Separation of horizontal and vertical scales

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla\left(\int_z^{\eta} \rho \, d\zeta + P_A\right) + \nabla\cdot(\mu\nabla\mathbf{u}) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \mathbf{u}}{\partial z}\right) - f\mathbf{k}\times\mathbf{u}$$

$$\frac{D\mathbf{u}}{Dt} = -g\nabla\eta + \frac{\partial}{\partial z}\left(\nu\frac{\partial \mathbf{u}}{\partial z}\right) - f\mathbf{k}\times\mathbf{u} + \alpha g\nabla\hat{\psi} - \frac{1}{\rho_0}\nabla p_A - \frac{g}{\rho_0}\int_z^{\eta}\nabla\rho \, d\zeta + \nabla\cdot(\mu\nabla\mathbf{u})$$

Boussinesq assumption (=>incompressibility)

$$\nabla\int_z^{\eta}\rho \, d\zeta = \int_z^{\eta}\nabla\rho \, d\zeta + \rho\nabla\eta$$

Momentum equation: vertical boundary condition (b.c.)

Surface $\nu \frac{\partial \mathbf{u}}{\partial z} = \boldsymbol{\tau}_w$ at $z = \eta$ Bottom $\nu \frac{\partial \mathbf{u}}{\partial z} = C_D |\mathbf{u}_b| \mathbf{u}_b$, at $z = -h$

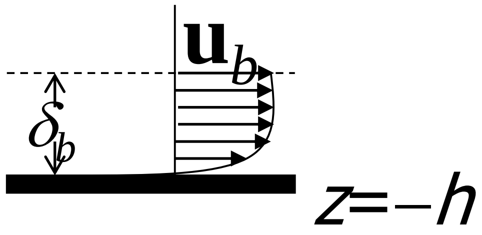
- Logarithmic law:

$$\mathbf{u} = \frac{\ln[(z+h)/z_0]}{\ln(\delta_b / z_0)} \mathbf{u}_b, \quad (z_0 - h \le z \le \delta_b - h)$$

z_0 : bottom roughness

$$\nu \frac{\partial \mathbf{u}}{\partial z} = \frac{\nu}{(z+h)\ln(\delta_b / z_0)} \mathbf{u}_b$$

- Reynolds stress:



$$\nu = \sqrt{2} s_m K^{1/2} l,$$

- Turbulence closure:

$$s_m = g_2,$$

$$K = \frac{1}{2} B_1^{2/3} C_D |\mathbf{u}_b|^2$$

$$l = \kappa_0 (z+h)$$

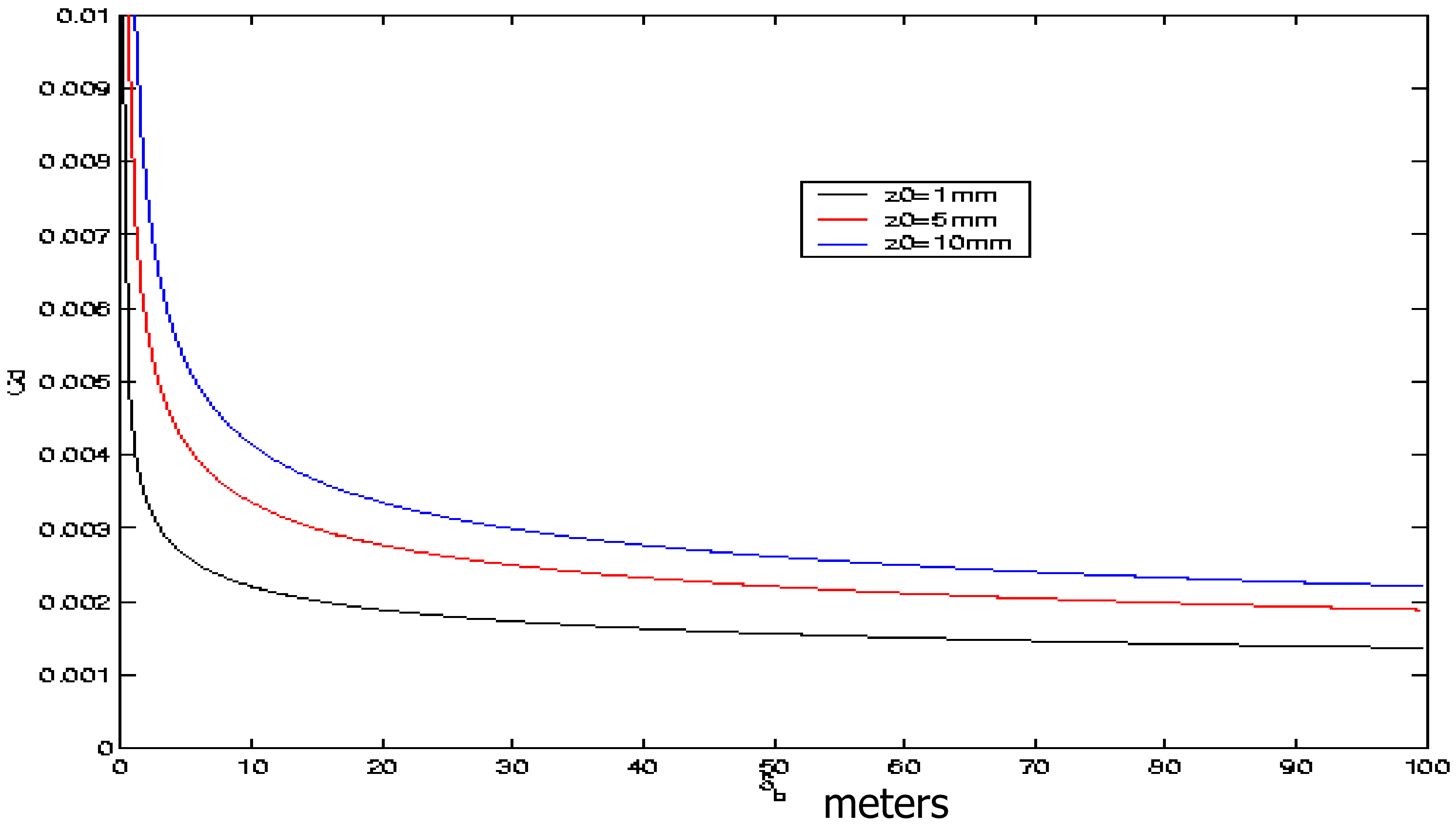
- Reynolds stress (const.)

$$\nu \frac{\partial \mathbf{u}}{\partial z} = \frac{\kappa_0}{\ln(\delta_b / z_0)} C_D^{1/2} |\mathbf{u}_b| \mathbf{u}_b, \quad (z_0 - h \le z \le \delta_b - h)$$

- Drag coefficient:

$$C_D = \left(\frac{1}{\kappa_0} \ln \frac{\delta_b}{z_0} \right)^{-2}$$

Bottom drag



- Use of z_0 seems most natural option, but tends to over-estimate C_D in shallow area
- C_D should vary with bottom layer thickness (i.e. vertical grid)
- Different from 2D, 3D results of elevation depend on vertical grid (with constant C_D)
- Simplest option for compound flooding study is to use variable Manning's n (2D)

Wind Shear Stress: Turbulent Flux of Momentum

Calculation of shear stress follows naturally from calculation of turbulent heat fluxes

Total shear stress of atmosphere upon surface:

$$\tau_w = \frac{\rho_a}{\rho_0} u_*^2$$

Alternatively, Pond and Picard's formulation can be used as a simpler option

$$\boldsymbol{\tau}_w = \frac{\rho_a}{\rho_0} C_{ds} \mid \mathbf{u}_w \mid \mathbf{u}_w$$

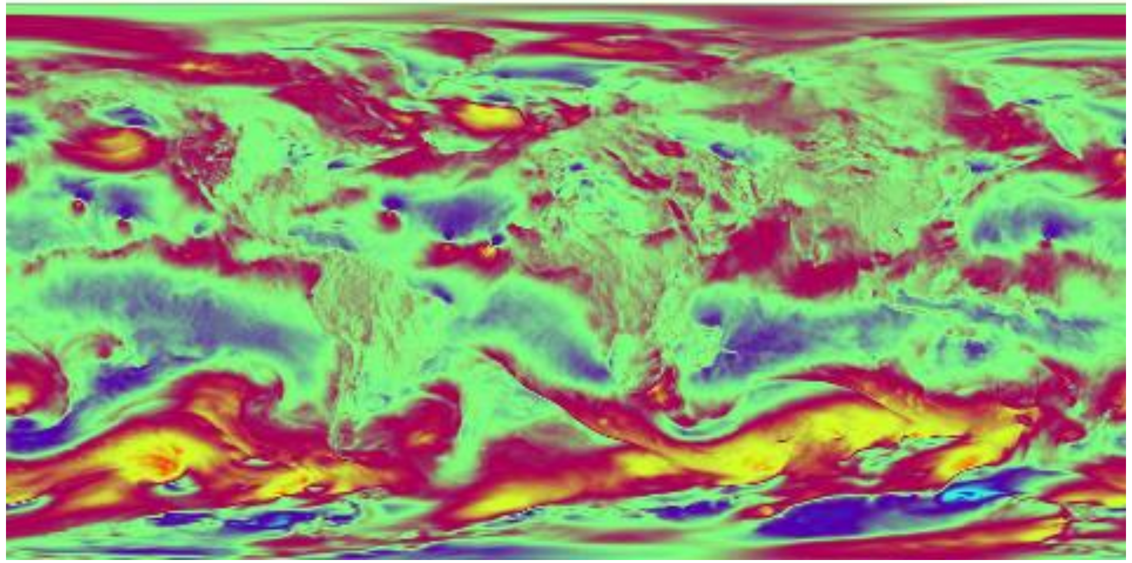
$$C_{ds} = \frac{0.61 + 0.063 u_w^{'}}{1000}$$

$$u_w^{'} = \max(6, \min(50, u_w))$$

Atmospheric inputs to air-sea exchange module

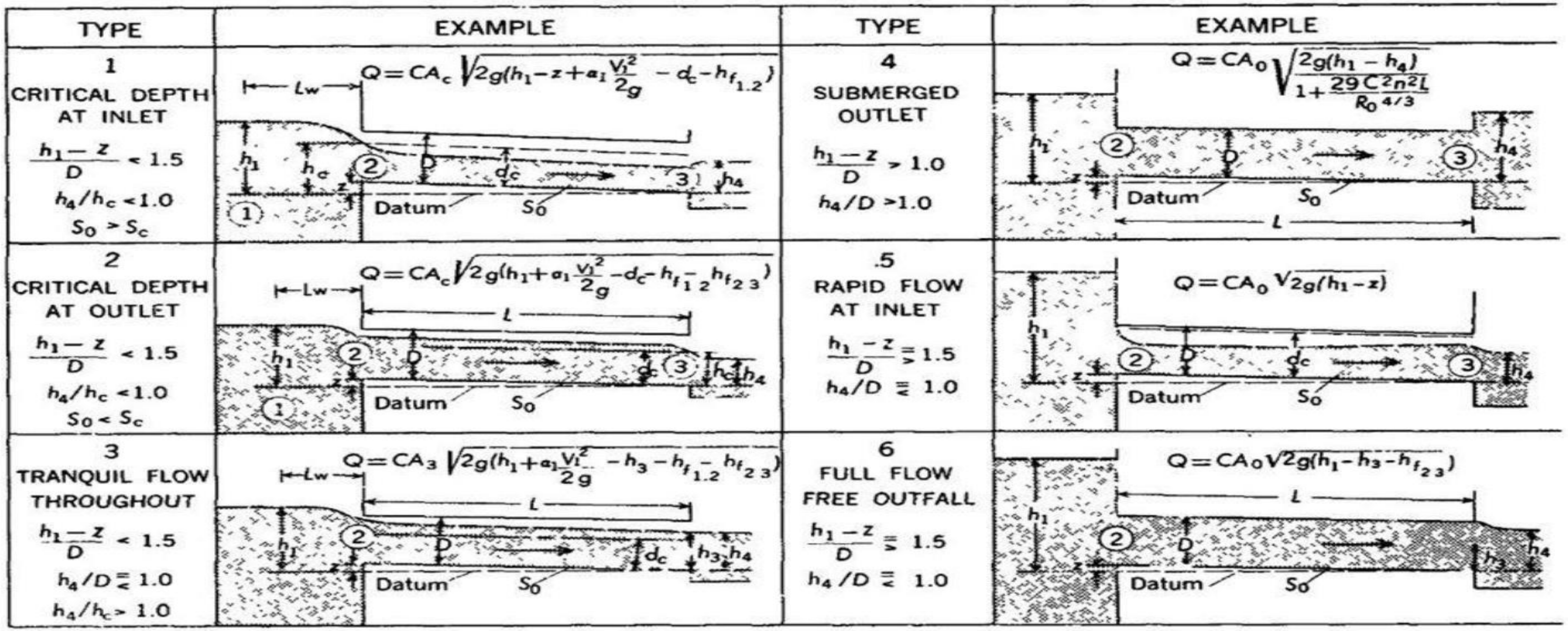
- Forecasts of u_w v_w @ 10m, P_A @ MSL, T_{air} and q_{air} @2m
  - used by the bulk aerodynamic model to calculate the scaling parameters
  - T_{air} and q_{air} not used in 2D; so set a constant for these
- Precipitation (sflux_prc*.nc): must be converted to source/sink for 2D config
- We have scripts to convert serval products into out sflux inputs (CF1.0 convention; structured grids)
  - CFSR
  - ERA
  - GFS
- Atmospheric forcing is also passed onto wave module (WWM) when invoked

Global ERA5 (u-wind)



Hydraulics

Classification of six types of culvert flow



TYPE	EXAMPLE	TYPE	EXAMPLE
1 CRITICAL DEPTH AT INLET $\frac{h_1 - z}{D} < 1.5$ $h_4/h_c < 1.0$ $S_0 > S_c$	$Q = CA_c\sqrt{2g(h_1 - z + \alpha_1 \frac{V_1^2}{2g} - d_c - h_{f_{1,2}})}$	4 SUBMERGED OUTLET $\frac{h_1 - z}{D} > 1.0$ $h_4/D > 1.0$	$Q = CA_0\sqrt{\frac{2g(h_1 - h_4)}{1 + \frac{29C^2n^2L}{R_0^{4/3}}}}$
2 CRITICAL DEPTH AT OUTLET $\frac{h_1 - z}{D} < 1.5$ $h_4/h_c < 1.0$ $S_0 < S_c$	$Q = CA_c\sqrt{2g(h_1 + \alpha_1 \frac{V_1^2}{2g} - d_c - h_{f_{1,2}} - h_{f_{2,3}})}$	5 RAPID FLOW AT INLET $\frac{h_1 - z}{D} \geq 1.5$ $h_4/D \leq 1.0$	$Q = CA_0\sqrt{2g(h_1 - z)}$
3 TRANQUIL FLOW THROUGHOUT $\frac{h_1 - z}{D} < 1.5$ $h_4/D \leq 1.0$ $h_4/h_c > 1.0$	$Q = CA_3\sqrt{2g(h_1 + \alpha_1 \frac{V_1^2}{2g} - h_3 - h_{f_{1,2}} - h_{f_{2,3}})}$	6 FULL FLOW FREE OUTFALL $\frac{h_1 - z}{D} \geq 1.5$ $h_4/D \leq 1.0$	$Q = CA_0\sqrt{2g(h_1 - h_3 - h_{f_{2,3}})}$

There is a manual for hydraulics structures in SCHISM

Orifice equation

$$Q = CA\sqrt{2g(y_1 - y_3)}$$

where A = Area of the gate opening
C = Discharge coefficient (typically 0.8)